

in which

$$\begin{aligned}\gamma_4 &= \mu_1 \frac{S_1}{S_1 + S_4} + \mu_2 \frac{S_4}{S_1 + S_4}; \\ \gamma_3 &= \gamma_1 \frac{S_2}{S_2 + S_3} + \gamma_2 \frac{S_3}{S_2 + S_3}; \\ \gamma_2 &= \left( \frac{l_1}{\mu_1} + \frac{1-l_1}{\mu_2} \right)^{-1}, \quad \gamma_1 = \left( \frac{l_2}{\mu_1} + \frac{1-l_2}{\mu_2} \right)^{-1}.\end{aligned}$$

If the combined concentration is  $v_1 > 0.5$ , the subscripts 1 and 2 are interchanged in  $K_i$  and  $\mu_i$  in (10) and (11) and for  $v_i$  in Table 1.

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#### RADIAL ELECTRON DENSITY DISTRIBUTION IN PLASMA FLOW IN A COAXIAL HALL ACCELERATOR

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UDC 533.9.082

The values of the electron density determined from measurements of the Starr broadening of a hydrogen line are presented.

Plasma accelerators are employed in diverse areas of science and technology. To increase their operating efficiency it is necessary to study the physical processes occurring in the plasma jet. Information about these processes can be obtained from measurements of the temperature and particle density.

In this paper we present the results of measurements of the electron density in the plasma flow in a coaxial Hall accelerator; the principle of operation and the construction of the accelerator are described in [1, 2]. The electron density was determined in the section of the plasma jet located 130 mm from the cutoff of the nozzle of the accelerator; the flow rate of the working gas  $C_{\Sigma} = 10$  g/sec (8.5 g of air and 1.5 g of nitrogen), the magnetic

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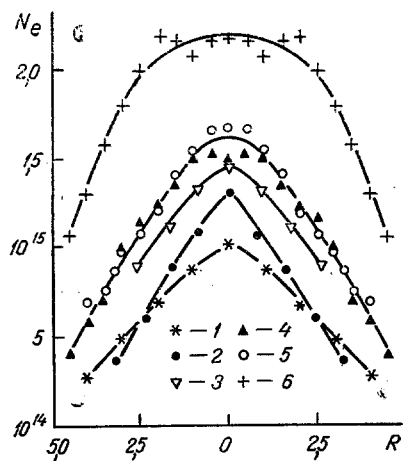


Fig. 1

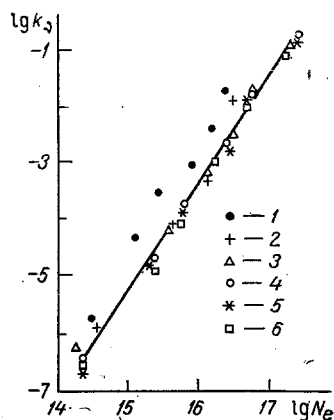


Fig. 2

Fig. 1. The distribution of the electron density over the radius of the jet for different discharge currents: 1)  $I = 2200$  A ( $l = 200 \mu\text{m}$ ); 2) 2200 (continuum background); 3) 2600 (continuum background); 4) 2600 (200) 5) 2600 (30); 6) 3000 (30).

Fig. 2. The absorption coefficient versus the electron density for different temperatures of hot air: 1)  $T = 10 \cdot 10^3$  K; 2)  $12 \cdot 10^3$ ; 3)  $14 \cdot 10^3$ ; 4)  $16 \cdot 10^3$ ; 5)  $18 \cdot 10^3$ ; 6)  $20 \cdot 10^3$ .

induction in the discharge zone  $B = 1$  T, and the current strength  $I = 2200, 2600,$  and  $3000$  A. A method based on measuring the broadening of the hydrogen line  $H_\beta$  owing to the linear Stark effect was used to determine the electron density. The Stark broadening of the  $H_\beta$  line is determined primarily by the electron density and is virtually independent of the temperature and the character of the electron velocity distribution, which makes this method applicable to nonequilibrium plasma [3].

The emission spectra of the plasma jet containing hydrogen as an additive were photographed with the input slit of the spectrograph 30 and  $200 \mu\text{m}$  wide. The procedure for photographing the spectra and performing photometric measurements on them is described in [1]. The contours of the  $H_\beta$  line, corresponding to different distances from the axial line of the plasma jet, were found by transferring from the integrated intensities at 15-20 monochromatic wavelengths, into which the wide  $H_\beta$  line was divided, to their local values with the help of Abel inversion [4]. The broadening of the  $H_\beta$  line in each separate case was determined from the contours of the lines taking into account the dispersion of the spectrograph.

The  $H_\beta$  line is also broadened due to the Doppler effect. The instrumental function of the spectrograph was also taken into account; its shape was assumed to be nearly Gaussian in the case of the  $30 \mu\text{m}$  slit and slit-like in the case of the  $200 \mu\text{m}$  slit. The Stark broadening was separated from the experimentally determined broadening under the assumption that the broadening of the central part of the  $H_\beta$  line is of an impact character [5].

The electron density  $N_e$  was determined from the graph of  $N_e$  versus  $\gamma_{st}$  presented in [3], or it was calculated using the formula [6]

$$N_e = 10^{13} \gamma_{st}^{3/2} [C_1(T) + C_2(T) \ln \gamma_{st}], \quad (1)$$

where  $C_1$  and  $C_2$  are constants, which are presented in [6]. Figure 1 shows the measurements of the electron density determined from spectra obtained with different slit widths of the spectrograph and processed with different slit widths of the microphotometer. It follows from the arrangement of points on this graph that for each separate distance from the axis of the source all values of the electron density are the same within the limits of error of the measurements, which we estimated to be 15-20%. This indicates that the reproducibility of the measurements is good.

As follows from Fig. 1, as the current strength is increased the electron density increases in both the central and peripheral regions. With a current strength of 2600 A, a region with virtually constant electron density over the cross section is observed in the part of the plasma flow near the axis. As the discharge current is increased this region increases in size, and with a current strength of 3000 A the cross-sectional area of the plasma flow, which is characterized by a constant electron density, now reaches 10-12  $\text{cm}^2$ . At the

same time the increase in the current strength does not appreciably affect the magnitude of the temperature gradient in the peripheral regions of the plasma flow.

The electron density was also determined from the intensity of the radiation in the continuous spectrum with wavelengths near 550 nm; in the case under study this region does not contain any molecular bands and wings of isolated lines. The method is based on comparing the experimentally determined radiation intensity with the theoretically computed intensity, which in general depends on the density of charged particles and the plasma temperature [5]. The calculation of the radiation intensity in the continuous spectrum is based on the absorption coefficients found theoretically for hot air taking into account induced emission [7]. The absorption coefficients in the continuous spectrum were calculated taking into account the main radiation processes at a fixed temperature in the pressure range 0.001-100 atm assuming Boltzmann and Maxwell distributions. The values of the electron density corresponding to the changing air pressure at different temperatures are tabulated in [8]. This makes it possible to establish the dependence of the absorption coefficient in the continuous spectrum on the electron density for any fixed temperature of hot air. We performed the corresponding rescaling for the region of the spectrum near 550 nm. The results of the calculations are presented in Fig. 2.

The radial distribution of the radiation intensity in the continuous spectrum for wavelengths near 550 nm in the plasma flows of a Hall accelerator was found from the experimentally measured values of the integrated intensities. The transformation from integral quantities to local quantities was performed by the method described in [4]. The brightness of the plasma layer was found using the formula [9].

$$B_{pl} = \tau(\lambda)\varepsilon(\lambda, T_{st})B(\lambda, T_{st}) \frac{I_{st}}{I_{pl}} \frac{l_{st}}{\Omega_{pl}} \frac{\Omega_{st}}{l_{st}} \quad (2)$$

SI-10-300 tungsten incandescent ribbon lamp was used as the standard source.

The measurements of the electron density from radiation in the continuous spectrum are presented in Fig. 2. The agreement between the measurements of  $N_e$  by the two different methods for determining  $N_e$  from the radiation in the continuous spectrum can be recommended for approximate calculations in flows of hot-air plasma, since such measurements are much less laborious than for the method based on the broadening of hydrogen lines.

The experimentally found values of the electron density and the previously obtained temperature of the plasma jet [1] make it possible to establish based on the existing criteria for LTE [5] that the plasma under study is a nonequilibrium plasma.

#### NOTATION

$I$ , discharge current, A;  $G_\Sigma$ , flow rate of the working gas, g/sec;  $B$ , magnetic induction, T;  $N_e$ , electron density,  $\text{cm}^{-3}$ ;  $H_\beta$ , Stark half-width, Å;  $T$ , temperature, K;  $I_{st}$  and  $I_{pl}$ , relative intensities of the standard and the plasma;  $t_{st}$  and  $t_{pl}$ , exposure times for the standard and the plasma, sec;  $l$ , slit width of the spectrograph,  $\mu\text{m}$ ;  $R$ , radius, cm;  $\Delta E$ , binding energy, eV;  $B_{st}$  and  $B_{pl}$ , brightness of the standard and the plasma layer,  $\text{W}/(\text{cm}^3 \cdot \text{sr} \cdot \text{deg})$ ;  $\tau$ , transmission coefficient of the window of the standard lamp;  $\varepsilon$ , black-body emissivity;  $\Omega_{st}$  and  $\Omega_{pl}$ , solid angle in the exposure of the standard and the plasma, sr;  $k_\nu$ , absorption coefficient,  $\text{cm}^{-1}$ . Indices:  $\Sigma$ , total flow rate;  $e$ , electron component;  $St$ , Stark broadening;  $pl$  and  $st$ , conditions for the plasma and the standard; and,  $\nu$ , frequency.

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## MATHEMATICAL MODELING OF EXPERIMENTS WITH THE HELP OF INVERSE PROBLEMS

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UDC 519.95:518.0

The problem of interpreting observations under conditions when the properties of the object of interest depend on the state of the object is studied. A procedure that permits studying an experiment from the viewpoint of achieving maximum information from it is proposed and justified on the basis of linear abstract models.

In the last few years the theory of the analysis of measurements based on the use of solutions of inverse problems has been developing rapidly. In thermal physics the well-known works [1-4] as well as many other investigations are devoted to this subject.

We shall analyze, based on the approach proposed by the theory of inverse problems, the information content of an experiment as a problem of determining the conditions under which the maximum information about an object can be obtained with a limited number of tests on the object and finite sampling of observations of the state of the object. In this connection we shall pose and study the following questions.

First, we shall determine the maximum volume of information that can be extracted from observations of a single function of state of the object. Second, we shall establish the nature of the experiment as well as where and how observations which will permit determining simultaneously a number of parameters characterizing the sought properties of the object should be performed. Finally, third, we shall determine how an experiment should be planned so that the required values can be determined with minimum error of identification taking into account the effect of a wide class of measurement noise and modeling errors.

We shall define the relation between the observed state  $u$ , the action on the object  $f$ , and the sought properties of the object  $a = \{a_k\}_{k=1, p}$  in the form of the following equation:

$$L_a u = f. \quad (1)$$

It is assumed everywhere below that Eq. (1) has a unique and stable solution  $u$  for fixed values of  $a$  and  $f$ , and it is also assumed that the domain  $D$  of the operator  $L_a$  does not depend on the values of  $a$  sought.

We shall obtain the answer to the first question assuming that the model (1) is given in the form of a superposition of commuting operators. In this case the following theorem holds.

**Theorem 1.** To determine all properties of an object described by the model (1), where

$L_a \equiv \sum_{k=1}^p a_k L_k$ ,  $a_k = \text{const}$ , it is necessary and sufficient to perform a single experiment in

which observations of the state  $u \in U_* = \{u_* : \sum_{k=1}^p \lambda_k L_k u_* = 0\}$ , where  $\lambda_k = \text{const}$ ,  $\exists i, j \in [1, p] : \lambda_i, \lambda_j$

$\neq 0$ . can be performed.

**Proof.** Necessity. Assume that among the states  $u \in D$  of the model (1) there exists a state  $u_*$  to which on a set of coefficients  $A$  there correspond nonunique values  $a' \neq a$ ". Subtracting from one another Eqs. (1) with these values of the coefficients we obtain

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Scientific-Production Union Kriogenmash, Balashikha. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 57, No. 3, pp. 494-500, September, 1989. Original article submitted April 5, 1988.